

# Generalized Weinberg's approach to the $a_0/f_0$ case

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**Abstract.** The problem of whether it is possible to distinguish composite from elementary particles is studied in the framework of Weinberg's approach. The possibility to extend this approach to the case of unstable particles in the presence of inelastic channels is considered. The interplay between the low-energy scattering data and the admixture of a bare state in the resonance is discussed, and the implications for the  $a_0(980)/f_0(980)$  case are outlined.

The nature of  $a_0(980)/f_0(980)$ -mesons remains the most enigmatic question of meson spectroscopy. Quark models [1] predict the  $1^3P_0$   $q\bar{q}$  states made of light quarks to exist at about 1 GeV. However, the vicinity of  $K\bar{K}$  threshold suggests that significant  $\frac{1}{\sqrt{2}}(u\bar{u} \pm d\bar{d})s\bar{s}$  admixture should be present in the wave functions of these mesons, either in the form of compact four-quark states [2] or in the form of  $K\bar{K}$  molecules [3], [4]. The related question to be addressed in this regard is of the relative role of  $s$ - and  $t$ -channel force in the formation of such molecules [5].

The approaches of [3], [6] and [4] conclude that the  $t$ -channel force can be dominant in producing the attraction in  $K\bar{K}$  channel necessary to form the  $a_0$  and  $f_0$  as hadronic molecules. On the other hand, as  $a_0/f_0$  couple strongly to  $K\bar{K}$  channel, one expects drastic unitarity effects, which are responsible for the dressing of the bare states seeded into mesonic continuum, the phenomenon described in the framework of coupled channel models [4], [7], [8].

Therefore the observed features of the  $\pi\pi$ ,  $\pi\eta$  and  $K\bar{K}$  spectra could be explained both by potential-type interaction in these systems and by existence of "bare" confined states strongly coupled to mesonic channels, and the question persists whether it is possible to distinguish between different assignments for  $a_0/f_0$ .

Many years ago S.Weinberg [9] has considered a similar problem of "elementarity" of the deuteron, expressing the effective range  $n$ - $p$  parameters in terms of field renormalization constant  $Z$ , which defines the admixture of a bare elementary-particle state in the deuteron. It was shown that the low-energy  $n$ - $p$  data are consistent with small value of  $Z$ , so that the deuteron is indeed a molecular-type particle made of proton and neutron.

To apply this approach, three requirements are needed. The particle should couple to a two-body channel with the threshold close to the nominal mass; this two-body channel should have zero orbital momentum; the particle must be stable, otherwise the factor  $Z$  is not defined. First two requirements are met in the  $a_0/f_0$  case, while the third one is not met: the decays  $f_0 \rightarrow \pi\pi$  and  $a_0 \rightarrow \pi\eta$  are known to be the main source of the width for these mesons. Nevertheless, it appears to be possible [10] to generalize Weinberg's

approach to the case of unstable particles.

The starting point of such generalization is the dynamical scheme of the coupled channel model. It is assumed that the hadronic state is represented symbolically as

$$|\Psi\rangle = \begin{pmatrix} \sum_{\alpha} c_{\alpha} |\psi_{\alpha}\rangle \\ \sum_i \chi_i |M_1(i)M_2(i)\rangle \end{pmatrix}, \quad (1)$$

where the index  $\alpha$  labels bare confined states  $|\psi_{\alpha}\rangle$  with the probability amplitude  $c_{\alpha}$ , and  $\chi_i$  is the wave function in the  $i$ -th two-meson channel  $|M_1(i)M_2(i)\rangle$ . The wave function  $|\Psi\rangle$  obeys the equation

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle, \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}_{MM} \end{pmatrix}, \quad (2)$$

where  $\hat{H}_c$  defines the discrete spectrum of bare states,  $\hat{H}_c|\psi_{\alpha}\rangle = E_{\alpha}|\psi_{\alpha}\rangle$ ,  $\hat{H}_{MM}$  includes the free-meson part as well as direct meson-meson interaction (e.g., due to  $t$ - or  $u$ -channel exchange forces), and the term  $\hat{V}$  is responsible for dressing the bare states. The latter is specified by the transition form factor  $f_{M_1(i)M_2(i)}^{\alpha}(p)$ ,

$$\langle \psi_{\alpha} | \hat{V} | M_1(i)M_2(i) \rangle = f_{M_1(i)M_2(i)M}^{\alpha}(p), \quad (3)$$

where  $p$  is the relative momentum in the mesonic system  $M_1(i)M_2(i)$ . The function  $f$  decreases with  $p$  with some range  $\beta$  whose scale is set by the size scale of hadronic wave functions; the estimate for  $\beta$  is to be of order of a few hundred MeV.

In a simple case of only one bare state  $|\psi_0\rangle$  and only one hadronic channel ( $|K\bar{K}\rangle$ ) the system of equations (2) is easily solved, yielding for the  $K\bar{K}$  scattering amplitude the form

$$F_{K\bar{K}}(k, k; E) = -\frac{2\pi^2 m f_{KK}^2(k)}{E - E_0 + g_K(E)}, \quad k = \sqrt{mE}, \quad (4)$$

where

$$g_K(E) = \int \frac{f_{KK}^2(p)}{\frac{p^2}{m} - E - i0} d^3 p. \quad (5)$$

If the system possesses a bound state with the energy  $-\varepsilon$ , the admixture of a bare state in the bound state wave function,  $|c_0|^2 = \cos^2 \theta$ , is defined from the expression

$$\tan^2 \theta = \int \frac{f_{KK}^2(p) d^3 p}{(\frac{p^2}{m} + \varepsilon)^2}. \quad (6)$$

In the small binding limit  $\sqrt{m\varepsilon} \ll \beta$  it is possible to express the effective range parameters in terms of the binding energy  $\varepsilon$  and angle  $\theta$  in a model-independent way (for the details see [10]). The relations between scattering length  $a$  and effective range  $r_e$ , and the binding energy  $\varepsilon$  and  $Z = \cos^2 \theta$  read

$$a = \frac{2(1-Z)}{2-Z} R + O(1/\beta), \quad r_e = -\frac{Z}{1-Z} R + O(1/\beta), \quad R = 1/\sqrt{m\varepsilon}, \quad (7)$$

coinciding with the ones obtained in [9].

In the case of unbound state one is to consider the continuum counterpart of  $Z$ , the spectral density of the bare state introduced in [11] and given by the expression

$$w(E) = 2\pi mk|c_0(E)|^2, \quad (8)$$

where  $c_0(E)$  is found from the system of Eqs. (2) in the continuum. Due to the normalization condition

$$\int_0^\infty w(E)dE = 1 - Z \text{ or } 1, \quad (9)$$

depending on whether there is a bound state or not, all the information on  $Z$  is encoded in the  $w(E)$  too, and the generalization to the multichannel case is straightforward.

If one is interested only in the phenomena near  $K\bar{K}$  threshold, it appears possible to express the spectral density  $w(E)$  in terms of hadronic observables. Indeed, one can make use of the smooth dependence on energy of the integral  $g_P$  for the light pseudoscalar channel, similar to (5). Then the near-threshold  $K\bar{K}$  scattering amplitude is given by the Flattè-type expression

$$F_{K\bar{K}} = -\frac{1}{2k} \frac{\Gamma_K}{E - E_f + i\frac{\Gamma_K}{2} + i\frac{\Gamma_P}{2}}, \quad (10)$$

where

$$\begin{aligned} E_f &= E_0 - \bar{E}_K - \bar{E}_P, \quad \Gamma_K = \bar{g}_{K\bar{K}}\sqrt{mE}, \quad \bar{g}_{K\bar{K}} = 4\pi^2 m f_{K0}^2, \\ \bar{E}_K &= 4\pi m \int_0^\infty f_{KK}^2(p)dp, \quad f_{K0} = f_{KK}(0), \end{aligned}$$

and  $\bar{E}_P$  and  $\frac{1}{2}\Gamma_P$  are the real and imaginary parts of the integral  $g_P$  averaged over  $K\bar{K}$  near-threshold region.

The spectral density can be written out as

$$w(E) = \frac{1}{2\pi} \frac{\Gamma_P + \bar{g}_{K\bar{K}}\sqrt{mE}\Theta(E)}{(E - E_f - \frac{1}{2}\bar{g}_{K\bar{K}}\sqrt{-mE}\Theta(-E))^2 + \frac{1}{4}(\Gamma_P + \bar{g}_{K\bar{K}}\sqrt{mE}\Theta(E))^2}. \quad (11)$$

Eq. (11) expresses the spectral density  $w(E)$  in terms of hadronic observables (Flattè parameters), just in the same way as Weinberg's factor  $Z$  is expressed in terms of hadronic observables (effective range parameters) via Eqs. (7). Thus, Eq. (11) generalizes Weinberg's result to the case of unstable particles.

It is clear from the expression (11) that it is the singularity structure of the scattering amplitude which governs the behaviour of spectral density. In the elastic case in the presence of a bound state the pole positions in the  $k$  plane are given by

$$k_1 = i\sqrt{m\varepsilon}, \quad k_2 = -i\sqrt{m\varepsilon} \frac{2-Z}{Z}. \quad (12)$$

For a deuteron-like situation, i.e. for  $Z \ll 1$ , the second pole is far from the threshold and even moves to infinity in the limit  $Z \rightarrow 0$ . On the other hand, if  $Z$  is close to one, i.e. if there is considerable admixture of an elementary state in the wave function of

**TABLE 1.** Pole positions (in MeV/c) and  $W$  for various fits to  $a_0$  (left) and  $f_0$  (right) mesons.

Ref.	$k_1$	$k_2$	$W_{a_0}$	Ref.	$k_1$	$k_2$	$W_{f_0}$
[13]	-104+i55	104-i111	0.49	[17]	-58+i107	58-i729	0.17
[14]	-134+i71	134-i199	0.29	[18]	-65+i97	65-i477	0.23
[15]	-129+i44	129-i250	0.24	[16]	-69+i100	69-i804	0.14
[15]	-126+i73	126-i212	0.29	[19]	-84+i17	84-i351	0.21
[16]	-102+i97	102-i199	0.36				

the bound state, both poles are near threshold. In the limiting case  $Z \rightarrow 1$  the poles are located equidistantly from the point  $k = 0$ . With inelasticity included, one expects the spectral density to be enhanced in the vicinity of the amplitude poles. If the poles are located in the near-threshold region, the spectral density in this region would be large. If, on the contrary, there is only one near-threshold pole, a considerable part of the spectral density is smeared over a much wider energy interval, which is a signal that the bare state admixture in the near-threshold resonance is small. So there is a one-to-one correspondence between the value of  $Z$  (or the behaviour of  $w(E)$ ) and the pole counting scheme suggested by Morgan [12].

Several Flattè-like fits to  $\pi\eta$  and  $\pi\pi$  spectra were analysed in [10], and the pole positions and near-threshold spectral densities for these fits were found. The results are given in Table 1 together with the integral

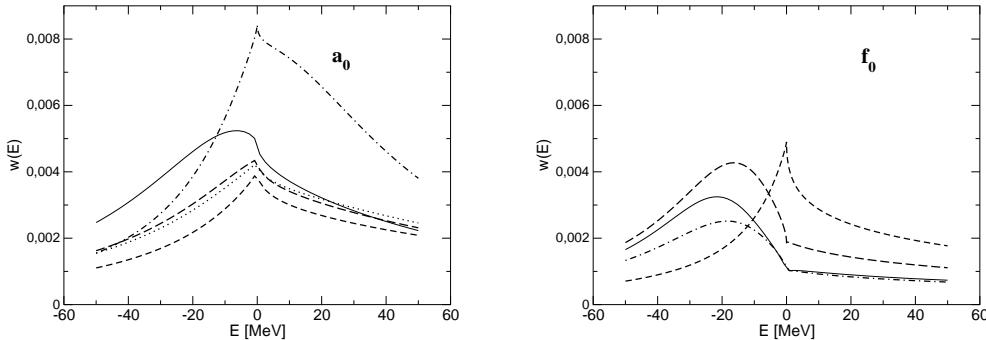
$$W_{a_0(f_0)} = \int_{-50MeV}^{50MeV} w_{a_0(f_0)}(E) dE . \quad (13)$$

of the spectral density over the region containing the  $K\bar{K}$  threshold. The limit of integration is chosen to be twice as large as the peak width of the  $a_0/f_0$  mesons. As the function  $w(E)$  is normalized to unity, the integral (13) is a direct measure of bare state admixture in the  $a_0/f_0$  mesons.

The spectral densities for various Flattè fits are shown at Fig.1. The  $a_0$ -meson looks like an above-threshold phenomenon, with considerable part of bare state spectral density peaked near  $K\bar{K}$  threshold. The  $f_0$  is a below-threshold resonance, with some small part of spectral density peaked below threshold. Obviously, this part can be viewed as Weinberg's "Z", smeared due to the presence of inelasticity, and this "Z" is definitely small.

The interrelation between pole positions and near-threshold fraction of the bare state spectral is clearly seen from the Table 1. The case of a pair of pole singularities near the threshold corresponds to the bare state accidentally seeded into near-threshold region; the admixture of a bare state should be large in this case. In the opposite case of small bare state admixture one has only one stable pole position near threshold.

Indeed, in the  $a_0$  case the fits lead to more equidistant positions of poles than in the  $f_0$  case, and the near-threshold fraction of  $w(E)$  is more sizable for the  $a_0$  meson. Still, even for  $a_0$ -meson it is, averagely, about 30%, so the  $a_0(980)$  contains a large admixture of mesonic components. As for the  $f_0$  meson, there is only one near-threshold pole, the near-threshold fraction of  $w(E)$  is about 20% or less, and mesonic component in  $f_0$  is large.



**FIGURE 1.** a) Spectral densities  $w(E)$  for the  $a_0$  meson based on the Flattè parameters taken from Ref. [13] (dashed-dotted line), Ref. [14] (dotted line), Ref. [15] (dashed line), Ref. [15] (long dashed line), and Ref. [16] (solid line). b) Spectral densities  $w(E)$  for the  $f_0$  meson based on the Flattè parameters taken from Ref. [17] (solid line), Ref. [18] (long-dashed line), Ref. [16] (dashed-dotted line), and Ref. [19] (dotted line).

We conclude, in such a way, that the simple  $q\bar{q}$  assignment is inadequate for the  $f_0(980)$ -meson, and is not very appropriate for the  $a_0(980)$ -meson.

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## REFERENCES

1. S. Godfrey, and N. Isgur, *Phys. Rev.* **D32**, 189 (1985).
2. R. L. Jaffe, *Phys. Rev.* **D15**, 267, 281 (1977); N. N. Achasov, S. A. Devyanin, G. N. Shestakov, *Phys. Lett.* **B96**, 168, (1980).
3. J. Weinstein and N. Isgur, *Phys. Rev.* **D27**, 588 (1979).
4. M. P. Locher, V. E. Markushin, and H. Q. Zheng, *Eur. Phys. J.* **C4**, 317 (1998).
5. N. Isgur and J. Speth, *Phys. Rev. Lett.*, **77**, 2332 (1996).
6. D. Lohse, J. W. Durso, K. Holinde, and J. Speth, *Phys. Lett.* **B234**, 235 (1990); G. Janssen, B. C. Pearce, K. Holinde, and J. Speth, *Phys. Rev.* **D52**, 2690 (1995).
7. E. van Beveren, C. Dullemond, and G. Rupp, *Phys. Rev.* **D21**, 772 (1980); G. Rupp, E. van Beveren, and M. D. Scadron, *Phys. Rev.* **D65**, 078501 (2002); E. van Beveren and G. Rupp, *hep-ph/0304105*.
8. N. A. Törnqvist, *Z. Phys.* **C68**, 674 (1995); N. A. Törnqvist and M. Roos, *Phys. Rev. Lett.* **76**, 1575 (1996).
9. S. Weinberg, *Phys. Rev.* **137** B672 (1965).
10. V. Baru, C. Hanhart, J. Haidenbauer, Yu. Kalashnikova, and A. Kudryavtsev, *hep-ph/0308129*.
11. L. N. Bogdanova, G. M. Hale, and V. E. Markushin, *Phys. Rev.* **C44**, 1289 (1991).
12. D. Morgan, *Nucl. Phys.* **A543**, 632 (1992).
13. S. Teige et al., *Phys. Rev.* **D59**, 012001 (2001).
14. D. V. Bugg, V. V. Anisovich, A. Sarantsev, and B. S. Zou, *Phys. Rev.* **D50**, 4412 (1994).
15. N. N. Achasov and A. N. Kiselev, *Phys. Rev.* **D68**, 014006 (2003).
16. A. Antonelli, eConf. C020620 THAT06 (2002).
17. M. N. Achasov et al., *Phys. Lett.* **B485**, 349 (2000).
18. R. R. Akhmetshin et al., *Phys. Lett.* **B462**, 380 (1999).
19. N. N. Achasov and V. V. Gubin, *Phys. Rev.* **D63**, 094007 (2001).